

CMB Polarization Systematics Due to Beam Asymmetry: Impact on Cosmological Birefringence

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The standard cosmological model is assumed to respect parity symmetry. Under this assumption the cross-correlations of the CMB's temperature anisotropy and 'gradient'-like polarization, with the 'curl'-like polarization identically vanish over the full sky. However, extensions of the standard model which allow for light scalar field or axion coupling to the electromagnetic field, or coupling to the Riemann gravitational field-strength, as well as other modifications of field theories, may induce a rotation of the CMB polarization plane on cosmological scales and manifest itself as nonvanishing TB and EB cross-correlations. Recently, the degree of parity violation (reflected in polarization rotation) was constrained using data from BOOMERANG, WMAP and QUAD. Forecasts have been made for near-future experiments (e.g. PLANCK) to further constrain parity- and Lorentz-violating terms in the fundamental interactions of nature. Here we consider a real-world effect induced by a class of telescope beam systematics which can mimic the rotation of polarization plane or otherwise induce nonvanishing TB and EB correlations. In particular, adopting the viewpoint that the primary target of future experiments will be the inflationary B-mode signal, we assume the beam-systematics of the upcoming PLANCK and POLARBEAR experiments are optimized towards this goal, and explore the implications of the allowed levels of beam systematics on the resulting precision of polarization-rotation measurements.

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INTRODUCTION

Future CMB polarimeters are poised to detect the B-mode polarization. It is expected that PLANCK will detect the lensing-induced B-mode in the near future and a host of CMB experiments are expected to detect the horizon-scale inflationary B-mode signal if the tensor-to-scalar ratio is $r \approx 0.01$. This extremely feeble signal is susceptible to a variety of foregrounds and systematics which may severely degrade the science which CMB observations could otherwise do in an ideal-world. Analytic expressions have been obtained for several types of beam systematics as well as the acceptable levels of beam uncertainties, such as differential ellipticity, gain, rotation, etc. (e.g. [1]-[4]). We are especially concerned with those beam features whose levels and other characteristics are not precisely known (e.g., the extent to which the beam is elliptical, its gain mismatch, etc) since this cannot be remedied in the process of data analysis and will therefore add an uncertainty to the CMB power spectra. The level of allowed beam uncertainty is determined mainly by the type of science one could do with B-mode polarization; these are associated with inflation and lensing-related science such as neutrino masses, dark energy equation of state, etc. It might be possible that an actual beam calibration will result in a better calibration than the minimum requirements for the science. As it turns out, for as low a tensor-to-scalar ratio as $r = 0.01$ the most stringent experimental requirements on the allowed level of beam

uncertainty come from the inflationary science. Not only will B-mode observations open a unique window to inflation and gravitational lensing, it also offers a rare opportunity to peer into two of the most fundamental underlying symmetries of physics, namely - parity and Lorentz invariance. With an horizon-scale optical path we have the longest possible leverage-arm to detect the accumulated effect of 'cosmic birefringence' (CB), a continuous rotation of the polarization plane of the electromagnetic radiation. Lorentz- and/or parity-violating terms in the electromagnetic or gravitational sectors of a hypothetical unified model of the fundamental interactions of nature may induce such a CB (e.g. [5]-[10]). Searching evidence for CB in the CMB power spectra have been the objective of several recent works ([11]-[16]) which analyzed the data in an ideal, systematics-free, world.

The purpose of this work is to highlight the importance of controlling beam systematics, mainly their effect on the polarization state of the CMB and in the context of CB. A tantalizing idea is to use the TB and EB cross-correlations in the CMB to detect Lorentz-symmetry breaking or parity-odd features. However, these cross-correlations are especially susceptible to even the smallest degradation in the B-mode (due to temperature leakage or beam rotation) and therefore a careful study of the implications beam systematics may have on the CB science is well motivated.

The outline of this paper is as follows. Section 2 summarizes CMB power spectra and relevant Fisher matrix relations. The effect of beam systematics is discussed in

section 3. We describe our analysis and results in section 4 followed by a discussion in section 5. We conclude in section 6. Our basic calculations are supplemented with two appendices: In Appendix A we estimate the resulting level of beam ellipticity assuming a standard procedure of telescope beam calibration (which determines the level of TB and EB correlations induced by beam ellipticity). Appendix B is dedicated to a brief discussion of the randomness of beam-pair orientation in the focal plane and its implications on the spurious power spectra.

CMB POWER SPECTRA AND CB

Before we describe the effect of non-standard physics on the CMB, let us first review the basics of the CMB statistics in the *standard* cosmological model because detecting this physics relies on deviations from the standard statistics of the CMB. The temperature T and the Stokes parameters Q & U are expanded in scalar and rank ± 2 tensor spherical harmonics

$$\begin{aligned} T(\hat{n}) &= \sum_{l,m} a_{lm}^T Y_{lm}(\hat{n}) \\ (Q \pm iU)(\hat{n}) &= \sum_{l,m} a_{\pm 2,lm} \pm Y_{lm}(\hat{n}) \end{aligned} \quad (1)$$

where the E and B (scalar and pseudo-scalar, respectively) modes of polarization have expansion coefficients E_{lm} and B_{lm} which can be expressed in terms of the spin ± 2 coefficients

$$a_{\pm 2,lm} = a_{lm}^E \pm a_{lm}^B. \quad (2)$$

It is assumed, and well motivated by cosmological inflation, that the temperature anisotropy and polarization modes can be drawn from gaussian distributions fully characterized by vanishing means and standard deviations described by the power spectra. This implies that the ensemble averages of the expansion coefficients vanish

$$\langle a_{lm}^X \rangle = 0 \quad (3)$$

where X can assume either T , E or B and that the widths of the gaussians satisfy (by the assumption of statistical isotropy which is the multipole-space dual of real space isotropy)

$$\langle a_{l'm'}^{X*} a_{lm}^{X'} \rangle = C_l^{XX'} \delta_{ll'} \delta_{mm'} \quad (4)$$

and

$$C_l^{XX'} \equiv \frac{1}{2l+1} \sum_{m=-l}^l a_{lm}^{X*} a_{lm}^{X'}. \quad (5)$$

In the standard model E and B are pure parity states (even and odd, respectively). The correlation, over the

full sky, of the B-mode with either the temperature or E-mode polarization vanishes in this case. However, if the polarization plane is rotated for some reason (cosmological or otherwise), there will be a E-B mixing which will induce the ‘forbidden’ TB and EB power spectra (leakage from the underlying TE and EE cross-correlations, respectively). Let us see how this effect takes place. While rotation of polarization leaves the temperature invariant, the polarization state will be affected. Consider for example the flat-sky approximation, which holds for small angular scales only (and is fully applicable on beam scales), but enables a transparent derivation of the following relations between the original and observed degraded power spectra. In this approximation the Stokes Q and U parameters, which fully describe the polarization state of a linearly polarized radiation, can be expanded in Fourier space as follows

$$(Q \pm iU)(\mathbf{x}) = \frac{1}{2\pi} \int (E_1 \pm iB_1) e^{i\mathbf{l} \cdot \mathbf{x}} e^{\pm 2i(\phi_l - \phi_x)} d^2\mathbf{l}. \quad (6)$$

$Q \pm iU$ are pure spin ± 2 states and under a rotation by $-\gamma$ pick up phases $e^{\pm 2i\gamma}$. As seen from Eq.(6) this results in an E-B mixing

$$\begin{aligned} E' &= E \cos 2\gamma - B \sin 2\gamma \\ B' &= E \sin 2\gamma + B \cos 2\gamma \end{aligned} \quad (7)$$

and correlating with $T' = T$ we obtain the following power spectra

$$\begin{aligned} C_l'^{TE} &= C_l^{TE} \cos 2\gamma \\ C_l'^{TB} &= C_l^{TE} \sin 2\gamma \\ C_l'^{EE} &= C_l^{EE} \cos^2 2\gamma + C_l^{BB} \sin^2 2\gamma \\ C_l'^{BB} &= C_l^{EE} \sin^2 2\gamma + C_l^{BB} \cos^2 2\gamma \\ C_l'^{EB} &= \frac{1}{2} (C_l^{EE} - C_l^{BB}) \sin 4\gamma. \end{aligned} \quad (8)$$

Note that no assumption has been made here as to the origin of this rotation, namely whether or not it is of cosmological origin. In the literature γ is identified with the CB rotation angle (see e.g. [15] and [16]). We will see in the next section that, in practice, one has to account for other, non-cosmological effects which mimic cosmological polarization-rotation and may bias the results for the best-fit γ if not properly accounted for (or, to a lesser extent, change the uncertainty of the inferred rotation itself). The main feature of Eq.(8) is that the ‘forbidden’ C^{TB} and C^{EB} do not vanish in general, and it is this property of the power spectra which will be used (in the near-future, once we have high signal-to-noise measurements of TB and EB correlations) to infer the level of CB. Current weak upper limits on γ are derived from comparing C_l^{TT} with C_l^{TE} and C_l^{EE} , all of which are primarily sourced by density perturbations. In the absence of CB, C_l^{TT} directly fixes C_l^{TE} and C_l^{EE} and the latter add little new cosmological information in the standard model with no reionization. Any deficit in the latter is ascribed to E-B rotation.

THE EFFECT OF BEAM SYSTEMATICS

No CMB observation is perfect and is always confronted, at some level, by optical beam systematics. Beam mismatch can induce temperature leakage to polarization as well as mode-mixing between the E- and B-modes ([1], [17]). Temperature leakage to polarization is considered the most pernicious systematic due to the fact that the primordial temperature anisotropy is at least an order of magnitude larger than the E-mode and two-three orders of magnitude larger than the B-mode. Therefore, even a small level of temperature leakage to polarization significantly contaminates the signal. For CB science, however, we are most concerned with cross-polarization and we open this discussion with the effect of beam rotation which mixes E and B-modes ([1], [17]). Beam rotation by an angle ε trivially biases γ at this level which implies that the uncertainty on the inferred γ cannot be smaller than the bias induced by the uncertainty in beam orientation, ε . Threshold values for ε have been found for POLARBEAR, PLANCK and CMBPOL [3] based on the requirement that both inflation and lensing science are not degraded above certain level. However, this requirement does not necessarily protect the CB science from beam systematics.

CMB polarimeters such as PLANCK will have sufficiently high instrumental sensitivity to allow a statistically significant detection of C^{TB} (in a systematics-free world), the ‘smoking gun’ for CB. As will be shown below, PLANCK and POLARBEAR have the raw sensitivity to detect rotation angles as small as few arcminutes. Any beam rotation by a larger angle will wash out the cosmological rotation of the polarization plane if CB is small enough. In these high-sensitivity experiments high polarimetric fidelity is required.

In the following we briefly discuss what impact two other beam systematics have on CB. These two effects, due to beam ellipticity and pointing error, leak temperature to polarization and are not a simple mixing of polarization as is the case with beam rotation. Since these two offsets will have a unique spectral-dependence in Fourier space (the systematic B-mode power spectra due to differential pointing and ellipticity scale as $\propto l^2 C_l^T$ and $\propto l^4 C_l^T$, respectively) they can be *statistically* distinguished and separated (with some residual ‘noise’), in principle, from CB which inherits the spectral shape of C_l^{TE} . Eqs.(7) will not be preserved in this case and we do expect that the likelihood function will change its *shape* and not be only biased. Table I summarizes the effect of differential rotation, pointing and ellipticity studied in [17] on C_l^{TB} and C_l^{EB} . It should be mentioned here that two other beam systematics considered in [17] and [3], differential gain and beamwidth, do not contribute to C_l^{TB} and C_l^{EB} . They do, however, affect C_l^{TE} , C_l^{EE} and C_l^{BB} , but since our main goal here is to test the

susceptibility of C_l^{TB} and C_l^{EB} to beam systematics and the implications it has on the inferred γ we do not address the effects of differential gain and beamwidth in this work.

ANALYSIS AND RESULTS

An easy way to assess parameter uncertainty (γ , the CB rotation angle in our case) is to invoke the Fisher information-matrix approach (though not in the presence of significant bias, as discussed below). The elements of the Fisher matrix are defined as follows

$$F_{ij} = \frac{1}{2} \sum_l (2l+1) f_{sky} \text{Trace}[\mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial \lambda_i} \mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial \lambda_j}] \quad (9)$$

where the 4-D symmetric covariance matrix \mathbf{C}_l (per each multipole l) is defined as

$$\mathbf{C}_l \equiv \begin{pmatrix} C_l^{TT} & C_l^{TE} & C_l^{TB} & C_l^{Td} \\ C_l^{TE} & C_l^{EE} & C_l^{EB} & 0 \\ C_l^{TB} & C_l^{EB} & C_l^{BB} & 0 \\ C_l^{Td} & 0 & 0 & C_l^{dd} \end{pmatrix} \quad (10)$$

where C_l^{dd} and C_l^{Td} refer to the power spectra associated with the lensing deflection angle [3]. The partial derivatives are taken with respect to the parameters λ_i . Here we allow nonvanishing TB and EB correlations and include lensing extraction. The dimensionality of the Fisher matrix is $N \times N$ where N is the dimension of parameter space of the cosmological model in question. The 1σ error on the parameter λ_i is given by

$$\sigma_{\lambda_i} = \sqrt{(\mathbf{F}^{-1})_{ii}}. \quad (11)$$

The 1σ error obtained by this procedure only sets a lower limit on the actual error. In general, when parameter degeneracy is high, as well as in the presence of significant bias, one resorts to Monte Carlo Markov Chain (MCMC) analysis [3]. The terms C_l^{TB} and C_l^{EB} drop from the matrix \mathbf{C} (Eq. 10) in the standard model, but in general, especially here, they do not.

A small bias in a parameter λ_i compared to the statistical uncertainty, is given by [2,3]

$$\Delta\lambda_i = \langle \lambda_i^{obs} \rangle - \langle \lambda_i^{true} \rangle = \sum_j (\mathbf{F}^{-1})_{ij} B_j \quad (12)$$

where the bias vector \mathbf{B} can be written as

$$\mathbf{B} = \sum_l (\mathbf{C}_l^{sys})^t \Phi^{-1} \frac{\partial \mathbf{C}_l^{cmb}}{\partial \lambda_j}, \quad (13)$$

$\Phi_{ij} \equiv \text{cov}(C_l^i, C_l^j)$ and \mathbf{C}_l is a vector containing all six power spectra. Our analysis includes lensing reconstruction and is essentially a generalization of [3].

As pointed above already, the underlying assumption of this work is that the primary target of upcoming CMB experiments is the inflationary B-mode; lensing-induced B-mode as well as the B-mode polarization induced by the Chern-Simons-type interaction term are only secondary. In addition, it was shown already [3] that the inflationary signal is more prone to beam systematics than the lensing signal is and therefore it was assumed that optimizing the beam systematics to the former should be sufficient for the latter. Here, we use the analytic expressions for beam systematics found in [17] to determine the resulting bias in the inferred CB for both PLANCK and POLARBEAR. Before presenting the results of our Fisher-matrix analysis it is constructive to plot the systematic C_l^{TB} and C_l^{EB} . These are shown for both PLANCK and POLARBEAR in Figure 1. Black (dotted), blue (dashed) and yellow (dotted-dashed) curves correspond to the systematics induced by differential pointing, ellipticity and beam-rotation, respectively (note that the effect of pointing on the EB cross-correlation is negligible and falls below the scale shown). For all the effects we assumed worst-case scanning strategy and polarimeter orientation as will be discussed below and extensively discussed in [17] and [3]. Red (continuous) curves show levels of C_l^{TB} and C_l^{EB} which these experiments could have detected (obtained from the Fisher-matrix analysis) in the absence of systematics. We refer to them as ‘nominal CB’ detection. For comparison, the green (continuous) curve corresponds to a much larger rotation of the polarization plane that will result in C_l^{TB} and C_l^{EB} roughly three times larger than the largest systematics-induced TB and EB correlations. It is evident from the figure that realizing the nominal potential of PLANCK and POLARBEAR to detect CB via the TB and EB correlations will require a much better control of systematics than the level set by the inflationary-induced B-mode detection requirements, for example.

The results of our Fisher-matrix analysis are shown in Table II. Nominal uncertainties in γ (i.e. the statistical error in a systematics-free experiment) are compared with the bias induced by differential pointing and ellipticity, $\Delta\gamma$. The values for the differential pointing ρ and ellipticity e are adopted from [3], consistent with the assumption that PLANCK and POLARBEAR are optimized for inflation and lensing science. In the analysis we used all information from multipoles $l \leq 1200$. Figure 1 suggests that the bias decreases if l_{max} is smaller but this of course comes on the expense of statistical uncertainty. We repeated the analysis with lower l_{max} which indeed resulted in lower bias but even for l_{max} as small as 200 it was unacceptably large. The effect of beam rotation results in a pure bias in the inferred γ and is therefore trivial and not included in the table. Again, we find that beam calibration levels that suffice for the inflationary and lensing science are unfortunately insuf-

ficient for the CB science and this is the main conclusion of this paper. Especially worrisome in this context are beam rotation and differential ellipticity. Both systematics are independent of the scanning strategy and therefore cannot be easily mitigated. In addition, beam rotation induces both EB and TB correlations which have the same spectral shape in multipole-space as the CB; one cannot distinguish a beam rotation from CB. While it is clear from our analysis that the allowed ellipticity based on the inflation+lensing requirements may not be sufficient for CB purposes, beam calibration in a real experiment will result in a better control of ellipticity than the minimum allowed by inflation/lensing. We show in Appendix A that both PLANCK and POLARBEAR will benefit from a reasonable control of ellipticity which will allow beam ellipticities lower than the minimum required for inflationary and lensing science and will therefore result in an essentially unbiased CB detection from the E-B correlations of the CMB in case the nominal values of the respective γ are considered. However, TB is still prone, even in this ideal case, to beam ellipticity. Beam rotation remains the paramount concern, and of all types of systematics we identify it as the main obstacle towards CB detection; the nominal γ values are typically a factor ≈ 10 smaller than the allowed beam rotation (Table 2). Recently, it was shown how optimal estimators, which can filter spatially-dependent rotation, can be used to filter-out a spatially-dependent rotation due to the non-standard statistics it induces by inducing $l-l$ mode coupling [18]. However, the *constant* beam rotation we consider here cannot be distinguished from CB by this method.

The systematics induced by differential pointing depend, to first order, on the scanning strategy and for a non-ideal, yet uniform, scanning strategy we encapsulate the relevant information in the two scanning functions f_2 and f_3 ([17]) whose exact definitions are irrelevant to this discussion since we always assume the worst case scenario ([3]) $f_i = 2\pi$.

The cosmological model adopted in our numerical calculations closely follows the WMAP5 results [15]; the baryon, cold dark matter, and neutrino physical energy densities in critical density units $\Omega_b h^2 = 0.021$, $\Omega_c h^2 = 0.111$, $\Omega_\nu h^2 = 0.006$. The latter is equivalent to a total neutrino mass $M_\nu = \sum_{i=1}^3 m_{\nu,i} = 0.56\text{eV}$. Dark energy makes up the rest of the energy required for a spatially-flat universe. The Hubble constant, dark energy equation of state and helium fraction are, respectively, $H_0 = 70 \text{ km sec}^{-1}\text{Mpc}^{-1}$, $w = -1$ and $Y_{He} = 0.24$. h is the Hubble constant in 100 km/sec/Mpc units. The optical depth to reionization and its redshift are $\tau_{re} = 0.073$ and $z_{re} = 12$. The normalization of the primordial power spectrum was set to $A_s = 2.4 \times 10^{-9}$ and its power law index is $n_s = 0.947$.

DISCUSSION

CMB observations, especially those which will have the sensitivity and fidelity to detect the ‘forbidden’ power spectra C^{TB} and C^{EB} , will be able to set tight constraints on CB, a phenomena related to the bedrock of fundamental physics and its symmetries. Cosmology improves on terrestrial experiments in this context due to the long optical path lengths which enable the detection, in principle, of extremely small effects. However, since a statistically significant detection of CB heavily relies on measuring the TB and EB ‘smoking gun’ correlations, and due to the fact that these involve the sub- μK B-mode signal which is prone to numerous systematics, a credible detection of non-vanishing γ should account for these possible sources of confusion. We saw that beam-rotation, differential ellipticity, as well as differential pointing, gain and beamwidth (to a lesser extent) can bias the inferred γ but also change the uncertainty (in the case of differential pointing and ellipticity) due to their direct effect on the TB and EB correlations. While the differential pointing is very small due to its shallow l -dependence (compared to other systematics considered here) and the effect of beam ellipticity can be partially harnessed (Appendix A), it is the beam rotation effect which mainly contaminates CB. In particular, we have shown that CMB experiments which are optimized for inflationary and lensing science *may not* be adequate for the CB science that requires a much better control of beam rotation. The nominal γ -detection with PLANCK and POLARBEAR is 3.8 and 1.1 arcminutes, respectively. Beam rotation should therefore be controlled to 0.75 and 0.21 arcminutes if a 5σ detection of CB is required. This systematic effect cannot be mitigated by scanning strategy. The effect of differential pointing (which is subdominant to the effect of differential ellipticity) can be further suppressed by scanning strategy mitigation [17]. The differential-ellipticity, together with the beam rotation effect, seem the most pernicious for TB spectra (Fig. 1). We consistently assumed the worst-case-scenario as we did when we employed the constraints on C^{BB} by assuming the tilt of the polarization direction to the ellipse major axis is 45° [3] (see also Figure 2). However, an estimate of the real uncertainty on beam ellipticity (Appendix A) shows that, in practice, beam ellipticity should not be a major problem to TB measurements by PLANCK and POLARBEAR. The EB cross-correlations, however, should vanish if we take $\psi = 45^\circ = \theta$. Nevertheless, for the purpose of illustration, and in the plots only, we assumed $\psi = 22.5^\circ = \theta$ so as to maximize the EB correlations for the given beam parameters [3]. We further explored the effect of randomness of the angle ψ in a multi-pixel elliptical-beam experiment (Appendix B) and found that our worst-case-scenario is actually representative of an average multi-

pixel experiment. Similar considerations were also applied to beam rotation.

CONCLUSION

CB is an interesting ancillary science that future CMB polarimeters will target. We considered in detail two such experiments: PLANCK and POLARBEAR. Our past experience taught us that controlling beam systematics to a sufficiently high precision as to aim at the B-mode detection (in case the tensor-to-scalar ratio is $r \approx 0.01$) will also automatically guarantee a high fidelity lensing-induced B-mode measurement. However, as we show here, this minimum requirement will in general not suffice for a credible CB detection via the ‘forbidden’ TB and EB correlations. The likely reason for this is that while BB correlations are quadratic in the beam-imperfection parameters (e.g. $\propto e^2, \varepsilon^2$) the TB correlations are only linear in these *small* beam parameters (and the EB correlation is in general noisy). This implies that for a given ellipticity or beam rotation the fractional bias in the TB cross-correlations will be larger than the the corresponding fractional bias in the BB power spectrum by $O(1/e)$ and $O(1/\varepsilon)$, respectively.

Whereas the effect of differential pointing on the TB and EB power spectra, we showed, is negligible, the effect of ellipticity is larger and can compromise the CB science if only the minimal requirement of the inflation/lensing science is satisfied. However, we demonstrated (Appendix A) that in practice the expected level of ellipticity uncertainty is relatively small (if the beam is standardly calibrated against a bright point source). The most pernicious effect is due to beam rotation which precisely mimics BC, has the same l -dependence, and cannot be mitigated by idealizing the scanning strategy. Our conclusion is that while TB and EB are unique indicators for new-physics in principle, they can in practice be excited by imperfect beams and in order to realize the promising potential of the high-sensitivity and fine-resolution PLANCK and POLARBEAR at CB detection beam rotation has to be controlled to the sub-arcminute level. Unless beam rotation is controlled to the arcminute level, a conservative approach, which does not use C_l^{TB} and C_l^{EB} , may be more adequate for the CB science.

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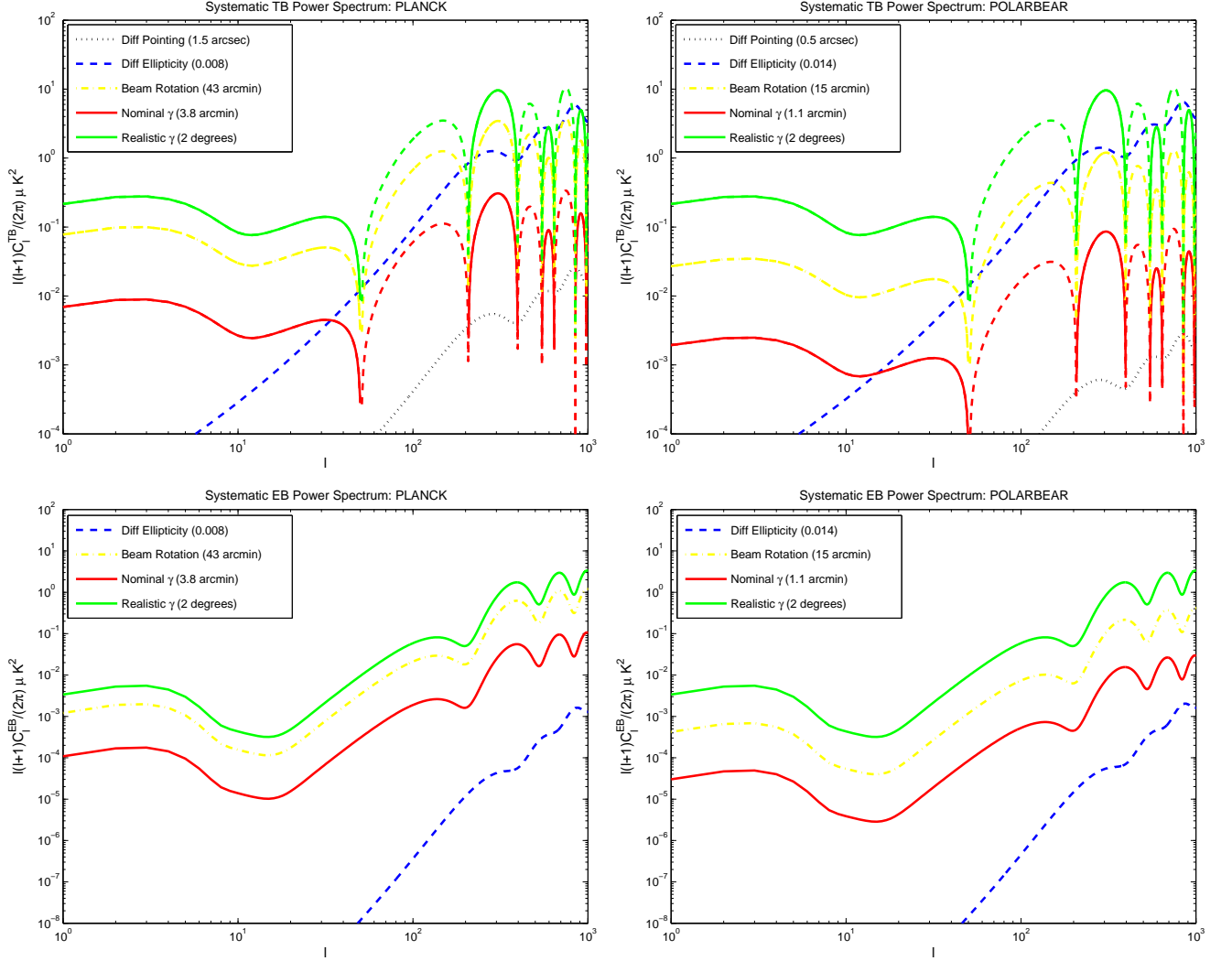


FIG. 1: TB and EB systematics compared to the nominal CB signals (i.e. those we expect to obtain in the absence of beam systematics): black (dotted), blue (dashed) and yellow (dotted-dashed) curves refer to the systematics induced by differential pointing and ellipticity and by beam rotation, respectively. Red and green (continuous) curves represent the cross-correlations in the nominal and realistic CB cases, respectively. ‘Realistic’ is defined as the CB that would be roughly three times larger than the largest systematic. Dashed-dotted curves represent the absolute values of negative T-B cross-correlations in certain l -ranges. All signals are calculated based on the assumption that the beam systematics of PLANCK and POLARBEAR are optimized for the detection of the inflationary signal at the $r = 0.01$ level [3]. These are used in the analytic expressions of [17] for C_l^{TB} and C_l^{EB} . It is clear from the upper panel that the main contaminant is due to differential ellipticity and beam rotation.

Appendix A: Forecasted Beam Ellipticity

It is instructive to estimate the expected uncertainty of beam ellipticity which may result from a standard beam-calibration procedure with a nearly black-body point-source such as Jupiter ($T_p \approx 150K$, $\theta_p \approx 0.5$ arcmin). By Wiener filtering [4] a map of the expected signal one expects to recover the source image with a signal-to-noise level

$$\left(\frac{S}{N}\right)^2 = \int \frac{|\tilde{S}(\mathbf{l})|^2}{P(\mathbf{l})} \frac{d^2\mathbf{l}}{(2\pi)^2}.$$

where $\tilde{S}(\mathbf{l})$ is the Fourier transform of the point source and $P(\mathbf{l})$ is the instrumental noise, i.e. the $(S/N)^2$ is the ratio of the signal and noise power-spectra integrated over all accessible multipoles in the experiment in question. The higher S/N the smaller are the uncertainties in the recovered beam parameters. The Fourier transform of the convolved calibration source is

$$\tilde{T}_p^{\text{obs}} = \tilde{T}_p e^{-\frac{1}{2}l_x^2\sigma_x^2 - \frac{1}{2}l_y^2\sigma_y^2 - il \cdot \rho} \quad (\text{A.2})$$

where we assume an elliptical gaussian beam with principal axes σ_x and σ_y , and pointing ρ . This results in

$$\left(\frac{S}{N}\right)^2 = \frac{2\ln(2)}{\pi(1-e^2)} \left(\frac{T_p}{\Delta_b}\right)^2 \left(\frac{\theta_p}{\theta_b}\right)^4 \eta^2 f_t \quad (\text{A.3})$$

where η is an experiment-specific optical-efficiency parameter (≤ 1) and f_t is the fraction of observation time dedicated to beam calibration (we adopt the value $1/30$). Δ_b is the instrumental temperature equivalent noise, θ_b is the beamsize and e is its ellipticity. Since the pointing merely adds a phase to the beam function it drops from the expression for S/N. Similarly, S/N is also independent of the beam rotation angle since temperature measurements are insensitive to ε . Calibrating the beam rotation with a polarized source will not work here since by analogy to Eq. (A.1) the rotation only adds a phase. Therefore, the following procedure, which is based on S/N considerations, will be used to determine the uncertainty of e only. To estimate these uncertainties we require that varying the beam parameters results in signal changes smaller than the noise. That is, we allow the uncertainty in the beam's ellipticity to grow from 0 until the $(S/N)^2$ changes by unity

$$(S/N)^2 \rightarrow (S/N)^2 + 1. \quad (\text{A.4})$$

This condition readily yields the uncertainty in beam parameters

$$\Delta(e^2) = [S\eta/N]^{-2} f_t^{-1}. \quad (\text{A.5})$$

Assuming Jupiter is a $150K$ black body with a characteristic scale of $\theta_p = 0.5'$, typical beamsizes θ_b are $5'$ ($4'$) and instrumental noise Δ_b $1.7\mu K$ ($10\mu K$) for PLANCK (POLARBEAR) as well as 5% optical efficiency η we obtain the following estimates on expected uncertainty in beam ellipticity $e = 1.1 \times 10^{-3}$ (PLANCK) and $e = 1.2 \times 10^{-4}$ (POLARBEAR). Using the scaling of ellipticity-induced TB and EB correlations ($\propto e$ and $\propto e^2$, respectively), it is clear that based on this estimate beam ellipticity is unlikely to be the dominant contaminant of the EB correlations but the TB correlations will still be somewhat biased even for the optimal ellipticity we find here ($O(10^{-3})$ for PLANCK and $O(10^{-4})$ for POLARBEAR).

Appendix B: Focal Plane Considerations

Previous works [1, 2, 3, 17] assumed, for simplicity, single-pixel experiments. However, in practice the beam-pairs are scattered in the focal plane with different polarization orientations and other beam parameters such as

gain, beamwidth, ellipticity, etc. We briefly discuss how the random orientation of beam polarization only mildly suppress spurious B-mode polarization. A schematic focal plane is depicted in Fig.2. We assume all beams have the same ellipticity for the purpose of illustration but they differ by their orientations with respect to the radius-vector which connects the center of the focal plane to the beam center (denoted χ) and the polarization axis makes an angle ψ with the ellipse major axis. We further assume that the major axes of the beams are nearly orthogonal ($\chi_1 - \chi_2 \approx 90^\circ$) and similarly their respective polarization directions satisfy ($\psi_1 - \psi_2 \approx 90^\circ$), where the subscripts stand for ‘beam 1’ and ‘beam 2’ of each pair, respectively. We calculate the covariance of the individual systematic power spectra

$$\text{cov}(C_l^Y, C_{l'}^Y) \equiv \delta_{l,l'} [(\langle C_l^Y \rangle^2) - (\langle C_l^Y \rangle)^2] \quad (\text{B.1})$$

where Y stands for either BB , TB or EB and the angular brackets denote averages over the angles ψ and χ . Since the number of pairs is finite, all the estimates we obtain here for the suppression of systematics by assuming infinitely many detector pairs constitute only upper limits. Nevertheless, we show that even if the number of detectors is assumed infinite, the gain in systematic suppression is insignificant. Taking the ratio of the covariance to the worst-case C_l^B , C_l^{TB} and C_l^{EB} ($\psi + \chi = 45^\circ$, 45° and 22.5° , respectively) it is straight forward to show that with an infinite number of pairs the systematic C_l^B , C_l^{TB} and C_l^{EB} drop to 71%, 71% and 50% of their worst-case values - an insignificantly small change to our conclusions. We can apply a similar calculation to beam rotation. in this case averaging is carried out over the angle ε (random orientations of the pair subject to the constraint that the polarization directions of the two polarimeters remain orthogonal). Here we obtain that C_l^B , C_l^{TB} and C_l^{EB} are suppressed to 35%, 71% and 50% of their worst-case values. Again, this will not quantitatively change our conclusions.

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effect	parameter	ΔC_l^{TB}	ΔC_l^{EB}
gain	g	0	0
monopole	μ	0	0
pointing	ρ	$\frac{1}{2}[1 + J_0(l\rho)]J_2(l\rho)s_\theta C_l^T$ $+ \frac{f_3}{2\pi}s_\theta J_1^2(l\rho)C_l^T$	$s_\theta c_\theta J_2^2(l\rho)C_l^T$
quadrupole	e	$-I_0(z)I_1(z)s_\psi C_l^T$	$I_1^2(z)s_\psi c_\psi C_l^T$
rotation	ε	$\sin(2\varepsilon)C_l^{TE}$	$\frac{1}{2}\sin(4\varepsilon)C_l^{EE}$

TABLE I: The contribution of the systematic effects to the power spectra C_l^{TB} , C_l^{EB} assuming the underlying sky is not polarized (except for the *rotation* signal when we assume E-, and B-mode polarization are present) and ideal sky scanning. The definitions of s_θ and s_ψ as well as f_3 can be found in [17] where similar expressions for C_l^{TE} , C_l^{EE} and C_l^{BB} are also found and which we employed in this work.

Experiment	$\sigma_\gamma^{\text{nominal}}$ [deg]	parameter	$\Delta\gamma$ [deg]
POLARBEAR	0.0179	ρ	0.0094
		e	56.07
PLANCK	0.0641	ρ	0.0060
		e	93.9

TABLE II: Estimated bias in the inferred rotation angle assuming POLARBEAR and PLANCK were optimized to detect the inflationary B-mode signal associated with $r=0.01$ ([3]). The bias induced by differential ellipticity is much larger than the naive uncertainty, highlighting the need in careful data analysis when interpretation of polarization plane rotation is required. The relatively small bias induced by the pointing is negligible compared to the ellipticity-induced bias and this is consistent with the findings of [3] and explained by the steep l -dependence of the ellipticity systematics.

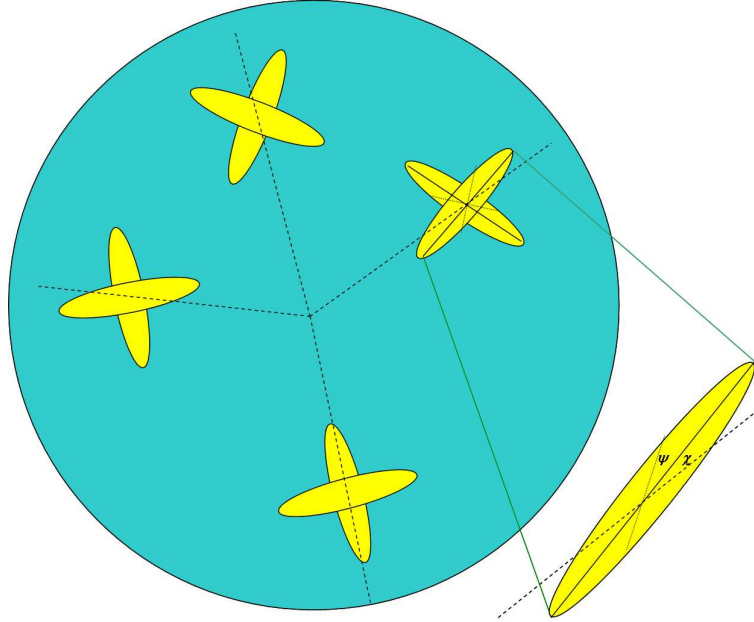


FIG. 2: Layout of typical beam pairs in the focal plane: dashed lines represent radius-vectors from the center of the focal plane through the center of each pair. Continuous lines represent the ellipse major axes and the angle the major axis makes with the radius-vector is χ . The polarization axis (dotted line) makes an angle ψ with the major axis. In [17] and [3] it was assumed that $\chi = 0$, therefore every ψ in our analytic expressions (Table 1) should be replaced with $\psi + \chi$.